Control Systems

Lecture: 4

Topics Covered

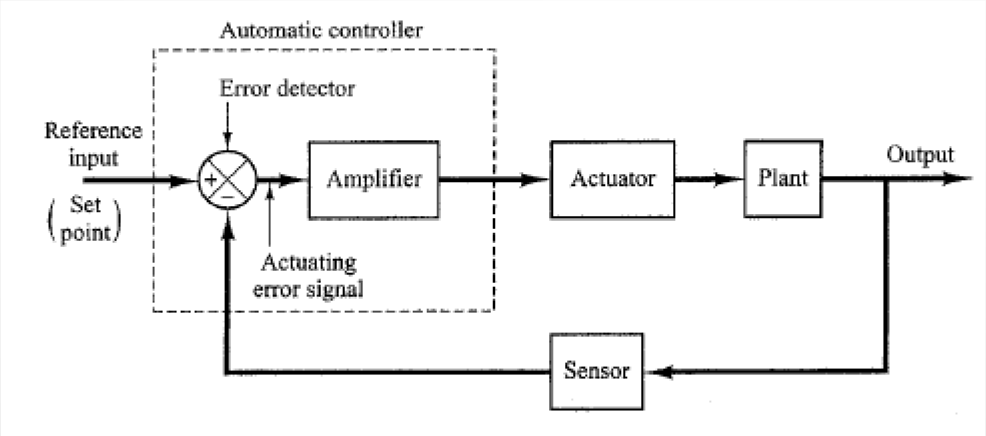
- Basic Control Action
- P, PI, PID controller

- A controller compares the actual value of

output with the reference input, determines the deviation, and produces a control signal that will reduce the deviation to zero or to a small value.

- The manner in which the controller produces the control signal is called the *control action.*

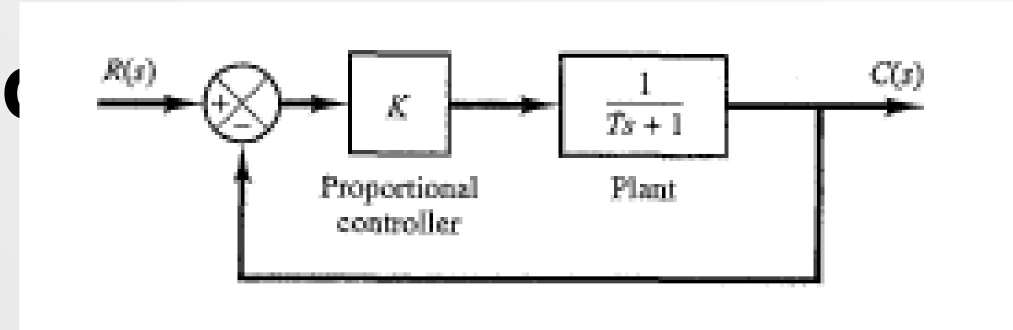
block diagram of an industrial control system



Classifications of Industrial Controllers.

- **1. Two-position or on-off controllers**
- **2. Proportional controllers**
- **3. Integral controllers**
- 4. Proportional-plus-integral controllers
- 5. Proportional-plus-derivative controllers
- ***6. Proportional-plus-integral-plus-derivative controllers***

Pr ... ystems.



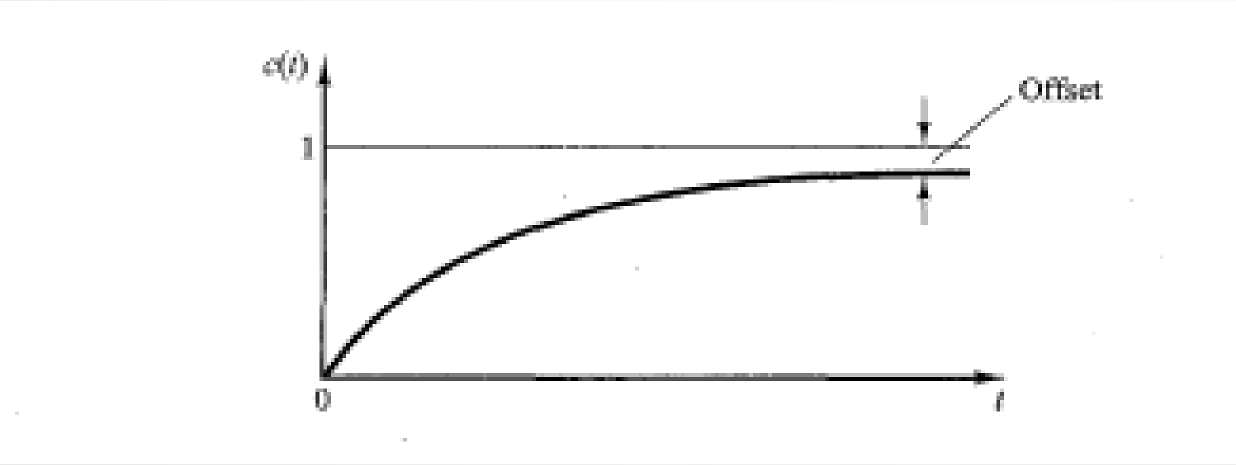
$$u(t) = K_p e(t)$$

$$\frac{U(s)}{E(s)} = K_p$$

$$\frac{E(s)}{R(s)} = \frac{R(s) - C(s)}{R(s)} = 1 - \frac{C(s)}{R(s)} = \frac{1}{1 + G(s)}$$

$$E(s) = \frac{Ts + 1}{Ts + 1 + K} \frac{1}{s}$$

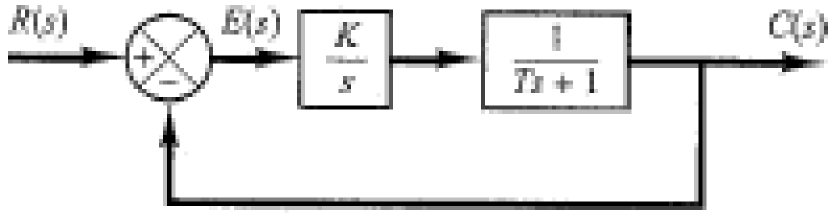
$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{Ts + 1}{Ts + 1 + K} = \frac{1}{K + 1}$$



ror in the step

l an offset.

tems.



$$e_{ss} = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} \frac{s^2(Ts+1)}{Ts^2+s+K}\frac{1}{s}$$

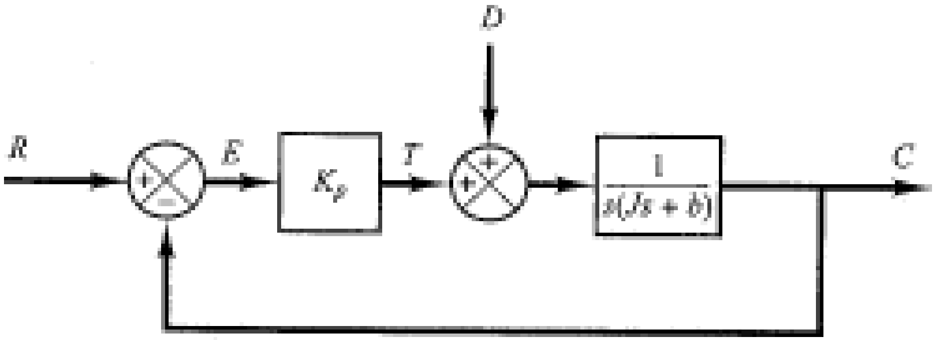
$$= 0$$

Integral control of the system eliminates the steady-state error in the response to the step input..

Response to Torque Disturbances (Proportional Control)

Assuming that the reference input is zero or $R(s) = 0$, the transfer function b
$C(s)$ and $D(s)$ is given by

$$\frac{C(s)}{D(s)} = \frac{1}{Js^2 + bs + K_p}$$



Hence

$$\frac{E(s)}{D(s)} = -\frac{C(s)}{D(s)} = -\frac{1}{Js^2 + bs + K_p}$$

The steady-state error due to a step disturbance torque of magnitude T_d is given by

$$e_{ss} = \lim_{s \to 0} sE(s)$$

$$= \lim_{s \to 0} \frac{-s}{Js^2 + bs + K_p} \frac{T_d}{s}$$

$$= -\frac{T_d}{K_p}$$

$$c_{ss} = -e_{ss} = \frac{T_d}{K_p}$$

Response to Torque Disturbances (Proportional-Plus-Integral Control

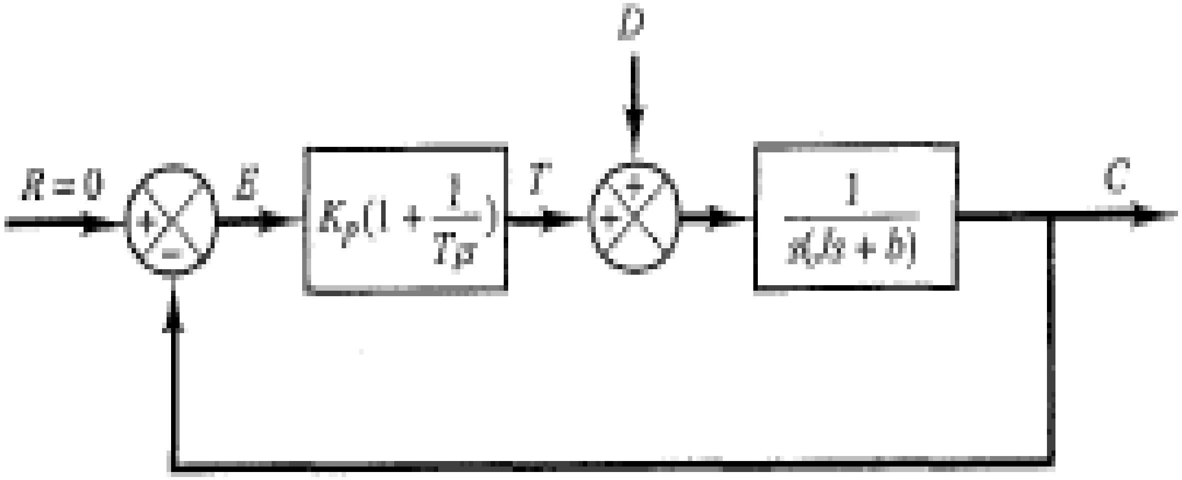
- **To** eliminate offset due to torque disturbance, the proportional controller may be replaced by a proportional-plus-integral controller.
- If integral control action is added to the controller, then, as long as there is an error signal, a torque is developed by the controller to reduce this error, provided the control system is a stable one.

The closed-loop transfer function between $C(s)$ and $D(s)$ is

$$\frac{C(s)}{D(s)} = \frac{s}{Js^3 + bs^2 + K_p s + \frac{K_p}{T_i}}$$

In the absence of the reference input, or $r(t) = 0$, the error signal is obtained from

$$E(s) = -\frac{s}{Js^3 + bs^2 + K_p s + \frac{K_p}{T_i}} D(s)$$



If this control system is stable, that is, if the roots of the characteristic equation

$$Js^3 + bs^2 + K_p s + \frac{K_p}{T_i} = 0$$

have negative real parts, then the steady-state error in the response to a unit-step disturbance torque can be obtained by applying the final-value theorem as follows:

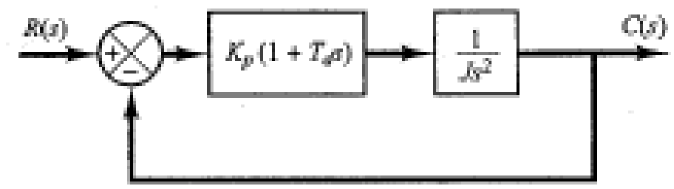
$$\begin{aligned} e_{ss} &= \lim_{s \to 0} sE(s) \\ &= \lim_{s \to 0} \frac{-s^2}{Js^3 + bs^2 + K_p s + \frac{K_p}{T_i}} \frac{1}{s} \\ &= 0 \end{aligned}$$

It is important to point out that if the controller was an integral controller, then the system always becomes unstable because the characteristic equation

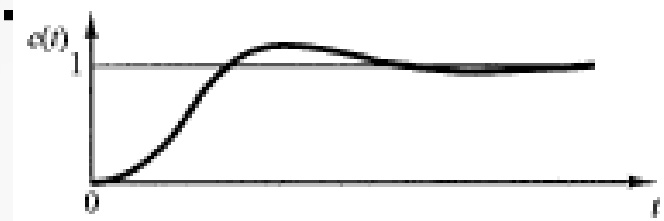
$$Js^3 + bs^2 + K = 0$$

will have roots with positive real parts. Such an unstable system cannot be used in practice.

Proportional-Plus-Derivative



(a)



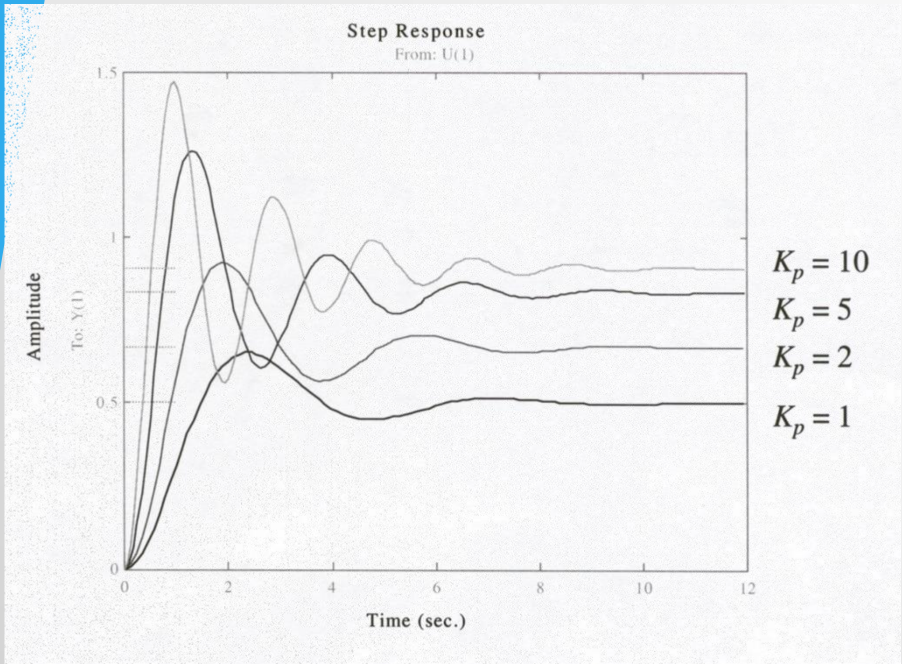
(b)

$$\frac{C(s)}{R(s)} = \frac{K_p(1 + T_d s)}{Js^2 + K_p T_d s + K_p}$$

. Thus derivative control introduces a damping effect. A typical response curve ***c (t) to a unit step*** input is shown in

- Effect of Proportional, Integral & Derivative Gains on the Dynamic Response

Change in gain in P controller



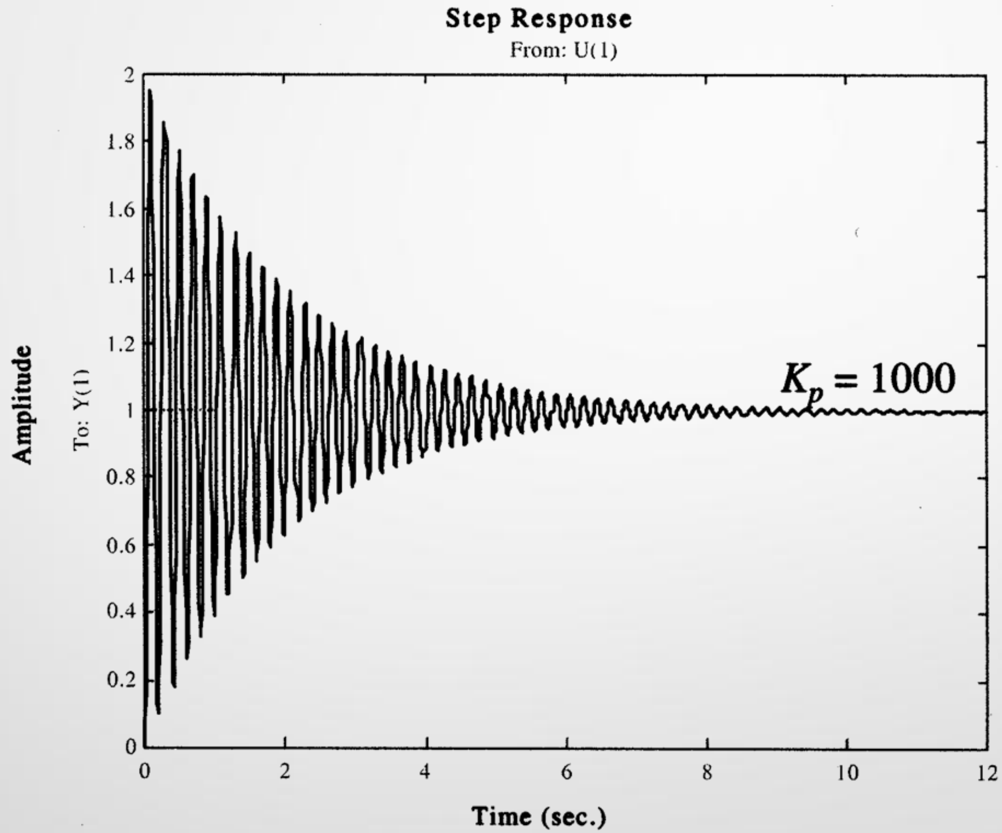
- **Increase in gain**:

  → Upgrade both steady-state and transient responses

  → Reduce steady-state error

  → ***Reduce stability!***

P Controller with ***high*** gain



Integral Controller

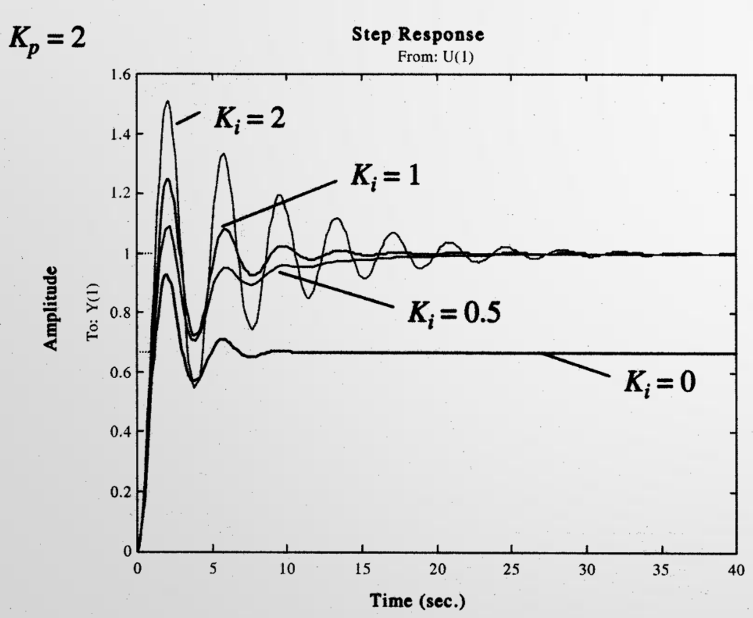
- Integral of error with a constant gain

→ increase the system type by 1

→*eliminate steady-state error for a unit step input*

→ amplify overshoot and oscillations

Change in gain for PI controller



- **Increase in gain**:

  → Do not upgrade steady-state responses

  → Increase slightly settling time

  → ***Increase oscillations and overshoot!***

Derivative Controller

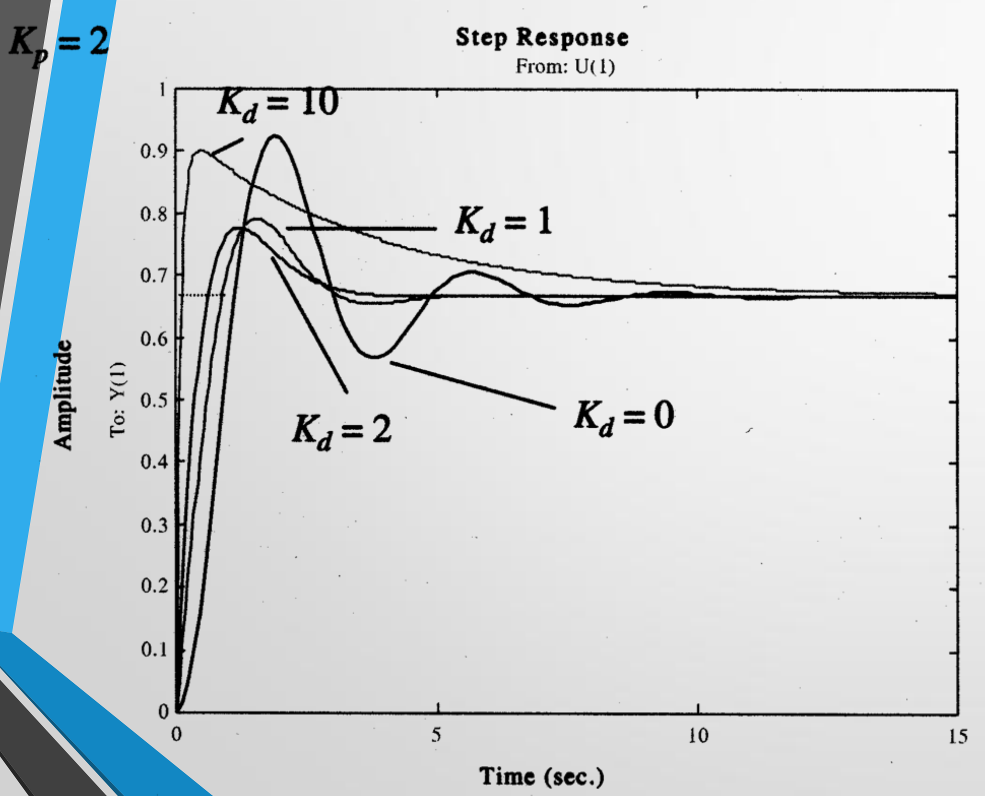
- Differentiation of error with a constant gain

→ detect rapid change in output

→ *reduce overshoot and oscillation*

→ do not affect the steady-state response

Effect of change for gain PD controller



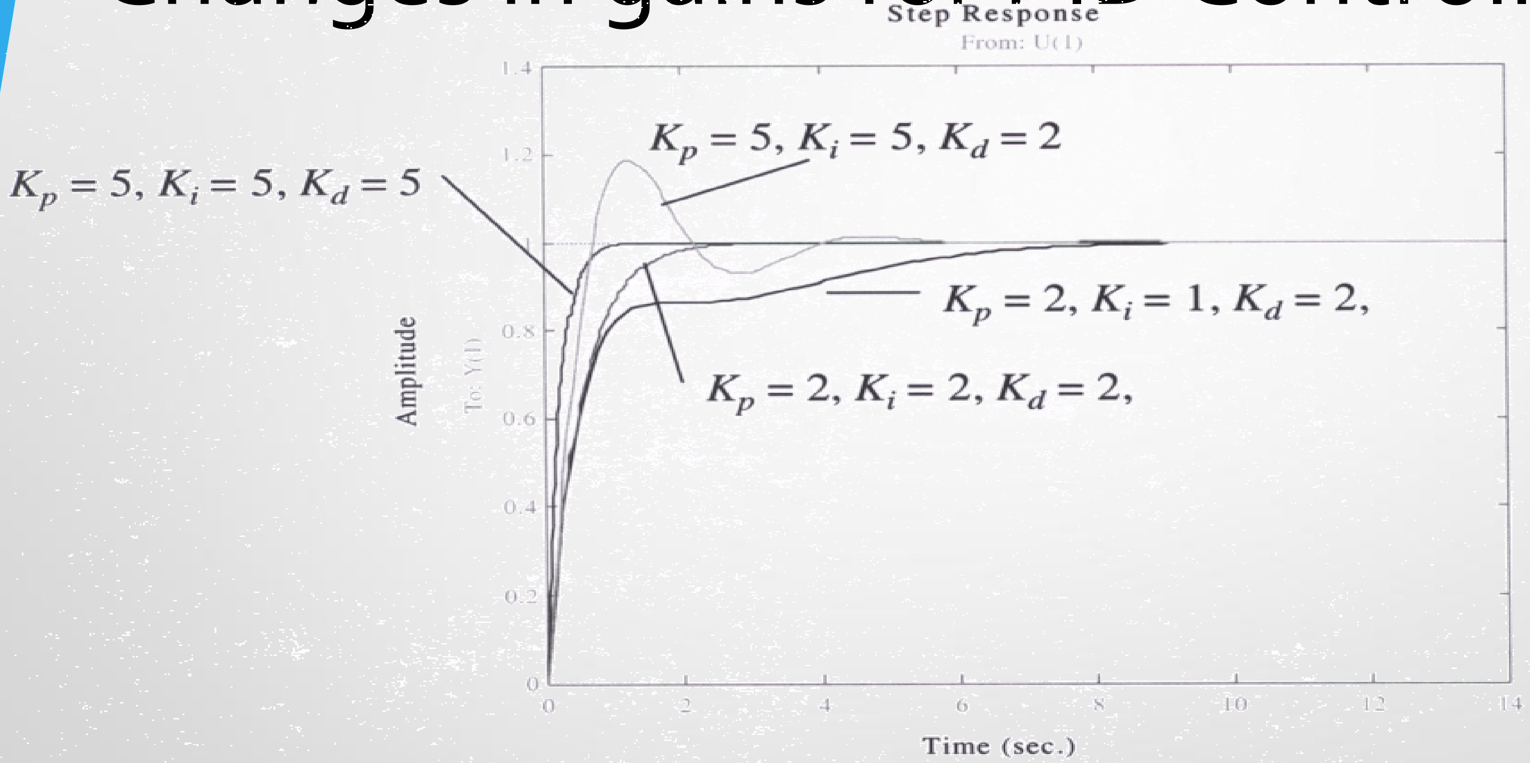
- **Increase in gain**:

  → Upgrade transient response

  → Decrease the peak and rise time

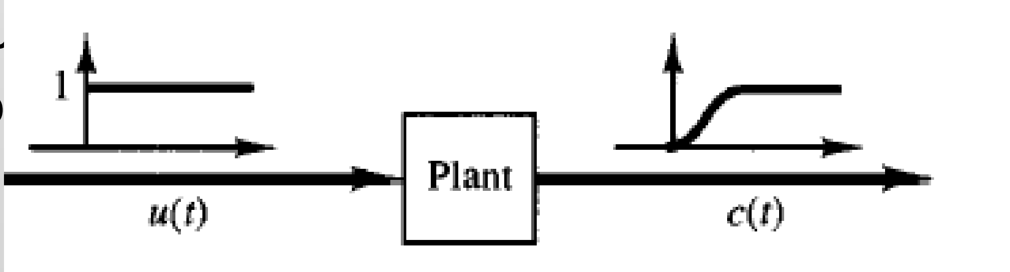
  → ***Increase overshoot and settling time!***

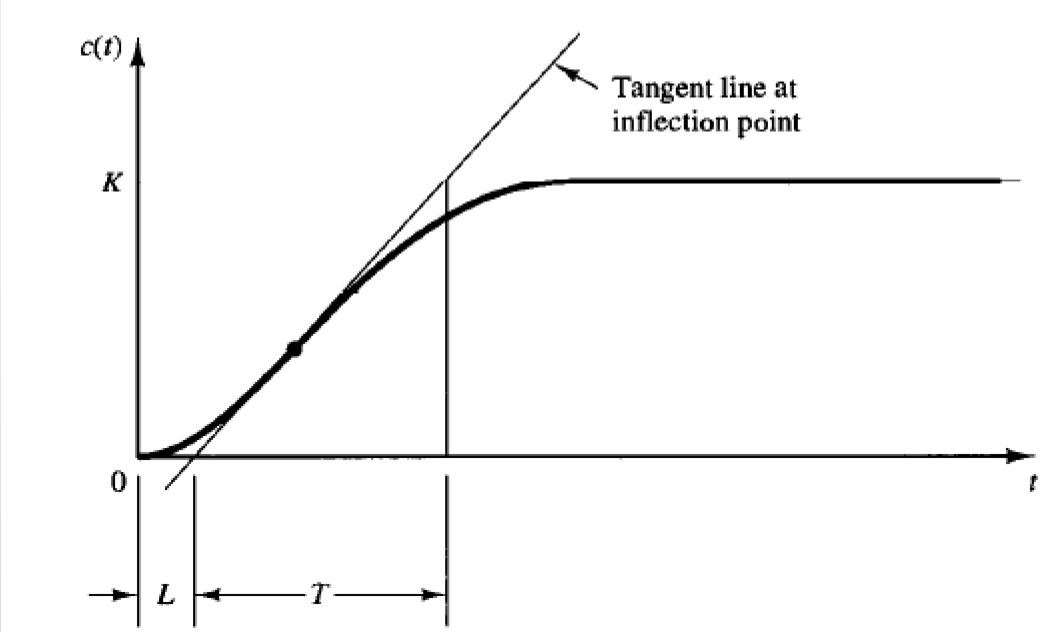
Changes in gains for PID Controller



Ziegler–Nichols rules for tuning PID controllers.

- These rules are used to determine Kp, Ti and Td for PID controllers
- First Method: The response is obtained experimentally to a u… …er integrators no…





c(t)
Tangent line at
inflection point
K
0
t
L
T

Ziegler and Nichols suggested to set the values of K_p, T_i, and T_d

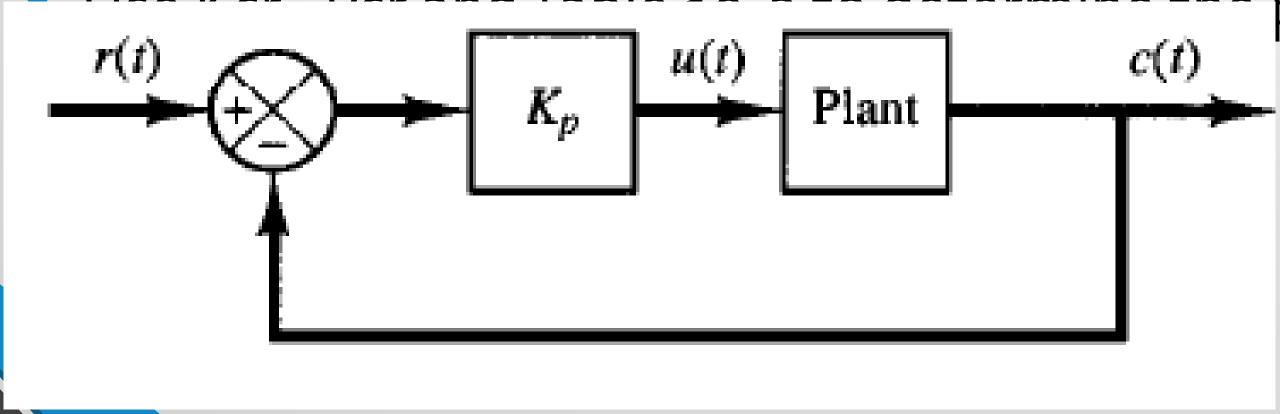
according to Table 10–1.

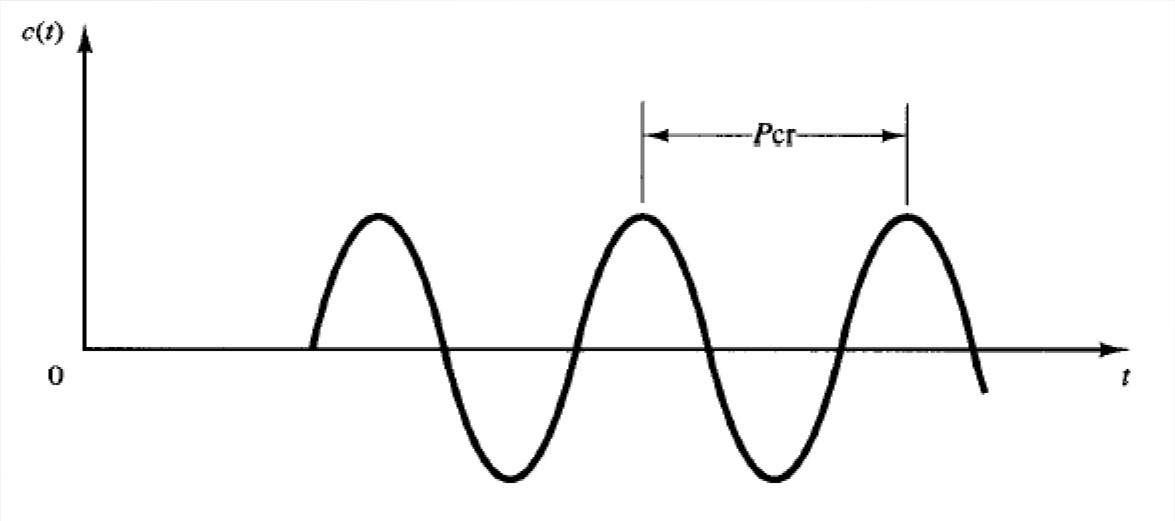
Table 10–1 Ziegler–Nichols Tuning Rule Based on Step Response of Plant (First Method)

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

Second Method

- Set Ti= inf and Td=0, increase Kp from 0 t a critical value Kcr where the output exhibits sustained oscillations.
- Use Kcr , Pcr and Table 10. 2 to determine the parameters





c(t)
Pcr
0
t

Table 10–2 Ziegler–Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period P_{cr} (Second Method)

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$